M E T U
Department of Mathematics

Group	Calculus II MidTerm 1	List No.

Code : *Math 120* Acad. Year : *2004-2005* Semester : *Spring* Instructor : Date : *April 2nd,2005* Time : *09:30* Duration : *120 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 5 QUESTIONS ON 6 PAGES TOTAL 50 POINTS

1	2	3	4	5	SOLUTION KEY	

Question 1 (10 points) Find the **radius** of convergence and **interval** of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

Solution:

$$\lim_{n\to\infty} \frac{\mid a_{n+1} \mid}{\mid a_n \mid} = \lim_{n\to\infty} \frac{3^{n+1} \mid x \mid^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{3^n \mid x \mid^n} = 3 \mid x \mid \lim_{n\to\infty} \sqrt{\frac{n+1}{n+2}} = 3 \mid x \mid .$$

So if $3 \mid x \mid< 1$, by ratio test the given power series is convergent.

This gives $\mid x \mid< 1/3$, so $R = 1/3$.

For the interval of convergence, we must check the end points.

If $x = 1/3$:

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{\sqrt{n+1}\, 3^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}.$$

$a_n = \frac{(-1)^n}{\sqrt{n+1}}$ is alternating,

- $\frac{1}{\sqrt{n+1}} > 0$,
- $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^n}{\sqrt{n+1}} = 0$,
- $\frac{|a_{n+1}|}{|a_n|} = \frac{\sqrt{n+1}}{\sqrt{n+2}} = \sqrt{\frac{n+1}{n+2}} < 1$, so $\mid a_n \mid$ is decreasing.

So by AST, the series is convergent.

If $x = -1/3$:

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{\sqrt{n+1}\,(-3)^n} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}}.$$

$\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ is divergent by $p-$test, so the above series is divergent by limit comparison.

So the interval of convergence is $I = (-1/3, 1/3]$.

Question 2 (10 points)

Consider $f(x) = \dfrac{1}{1+x^k}$; $k \geq 1$ an integer.

a) Find the power series expansion of $f(x)$ in powers of x.

Solution:

Since

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \quad \text{for all } \mid x \mid < 1,$$

we have;

$$\frac{1}{1+x^k} = \sum_{n=0}^{\infty} (-1)^n x^{nk}.$$

b) Use the series in part (a), to expand $g(x) = x \arctan(x)$ in powers of x.

Solution:

$$g(x) = x \arctan x = x \int_0^x \frac{1}{1+u^2} du = x \int_0^x \sum_{n=0}^{\infty} (-1)^n u^{2n} du = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{2n+1} \quad \text{for all } \mid x \mid < 1.$$

c) Find the expansion of $h(x) = \sqrt{x+1}\arctan(\sqrt{x+1})$ around $a = -1$. Indicate the interval of convergence.

Solution:

$$h(x) = \sqrt{x+1}\arctan\sqrt{x+1} = \sum_{n=0}^{\infty} (-1)^n \frac{\left[(x+1)^{1/2}\right]^{2n+2}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^{n+1}}{2n+1} \quad \text{for all } \mid x+1 \mid < 1.$$

Question 3 (9 points) Determine whether the followings are convergent or divergent. Show tour work.

a) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$

Solution:

We apply ratio test, with $a_n = \frac{(n!)^2}{(2n)!}$:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n!)^2(n+1)^2}{(2n)!(2n+1)(2n+2)} \cdot \frac{(2n)!}{(n!)^2} = \lim_{n\to\infty} \frac{n+1}{2(2n+1)} = \frac{1}{4} < 1.$$

So by ratio test, the series converges.

b) $\sum_{n=0}^{\infty} \frac{1}{2+3^n}$

Solution:

We have;

$$0 \leq \frac{1}{2+3^n} \leq \frac{1}{3^n} = \left(\frac{1}{3}\right)^n.$$

$\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$ is a geometric series with $r = 1/3$, so it is convergent.

Therefore, by comparison test, $\sum_{n=0}^{\infty} \frac{1}{2+3^n}$ is convergent.

c)Approximate $\frac{1}{e}$ with an error less than $\frac{1}{119}$.

Solution:

We have;

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}.$$

Then;

$$\frac{1}{e} = e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots.$$

Since the series is alternating;

$$E_n(1) = \frac{1}{(n+1)!} < \frac{1}{119},$$

which gives $n + 1 = 5$. Therefore;

$$\frac{1}{e} \approx 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} = \frac{3}{8}.$$

Question 4 (9 points)

a) Prove that for $a_n \geq 0$, if $\sum_{n=0}^{\infty} a_n$ is convergent then $\sum_{n=0}^{\infty} a_n^2$ is also convergent.

Solution:

Since $\sum_{n=0}^{\infty} a_n$ is convergent, we have $\lim_{n\to\infty} a_n = 0$. This implies that there exists $N \in \mathbb{N}$ such that for $n \geq N$, we have $0 \leq a_n \leq 1$. So for all $n \geq N$, $0 \leq a_n^2 \leq a_n$.

Since $\sum_{n=0}^{\infty} a_n$ is convergent, $\sum_{n=0}^{\infty} a_n^2$ is also convergent by comparison test.

b) The term of a sequence are defined recursively by the equations
$a_1 = 2, \quad a_{n+1} = (\frac{4n+1}{5n+2})a_n$. Is $(a_n)_{n=1}^{\infty}$ convergent? (explain, and if so what is the limit?)

Solution:

Consider $\sum_{n=1}^{\infty} a_n$. We have $a_n \geq 0$. Apply ratio test to this series;

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{4n+1}{5n+2} = \frac{4}{5} < 1.$$

So by ratio test, the series converges. This implies $\lim_{n\to\infty} a_n = 0$.

c) Give an example of a power series which is convergent in $(1,5)$, and divergent on $(-\infty,1) \cup (5,\infty)$. (No condition at $x=1$ and $x=5$)

Solution:

Consider, $\sum_{n=0}^{\infty} a_n(x-3)^n$; a power series around 3. We want the radius of convergence to be 2. So we want $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \frac{1}{2}$.

Let $a_n = \frac{1}{2^n}$, then we get the desired limit and interval of convergence. So

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n}$$

is a power series satisfying the conditions we want.

Question 5 (12 points)

a) Find all vectors $\vec{v}$ that satisfy the equation $(-\vec{i}+2\vec{j}+3\vec{k})\times\vec{v}=\vec{i}+5\vec{j}-3\vec{k}$

Solution:

Let $\vec{v}=(a,b,c)$. Then we should have;

$$(-1,2,3)\times(a,b,c)=\begin{vmatrix} i & j & k \\ -1 & 2 & 3 \\ a & b & c \end{vmatrix}=(2c-3b,3a+c,-2a-b)=(1,5,-3).$$

This happens if and only if $-3b+2c=1$, $3a+c=5$, and $-2a-b=-3$ which gives

$$\vec{v}=t\,\vec{i}+(3-2t)\,\vec{j}+(5-3t)\,\vec{k}\quad \text{for all } t\in\mathbb{R}.$$

b)Show that the four points $A(1,2,-1)$, $B(0,1,5)$, $C(-1,2,1)$ and $D(2,1,3)$ are coplanar (i.e. are on the same plane).

Solution:

Let the three vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ represent the three segments $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$, respectively. Then $\vec{u}=(-1,-1,6)$, $\vec{v}=(-2,0,2)$ and $\vec{w}=(1,-1,4)$.

The four points are coplanar if the volume of the parallelpiped spanned by the three vectors is 0;

$$\begin{vmatrix} -1 & -1 & 6 \\ -2 & 0 & 2 \\ 1 & -1 & 4 \end{vmatrix}=-\begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix}+\begin{vmatrix} -2 & 2 \\ 1 & -1 \end{vmatrix}+6\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}=-2-10+12=0.$$

c)Find the distance between the lines
$\mathcal{L}_1$: intersection of the planes ($x + 2y = 3$ and $y + 2z = 3$) and
$\mathcal{L}_2$: intersection of the planes ($x + y + z = 6$ and $x - 2z = -5$)

Solution:

$\mathcal{L}_1$ contains the points $(1, 1, 1)$, $(3, 0, 3/2)$, so it is parallel to the vector $(3, 0, 3/2) - (1, 1, 1) = (2, -1, 1/2)$, so it is parallel to the vector $\vec{u}_1 = (4, -2, 1)$.

$\mathcal{L}_2$ contains the points $(-5, 11, 0)$, $(-1, 5, 2)$, so it is parallel to the vector $(-1, 5, 2) - (-5, 11, 0) = (4, -6, 2)$, so it is parallel to the vector $\vec{u}_2 = (2, -3, 1)$.

Using the vectors $\vec{r}_1 = (1, 1, 1)$, $\vec{r}_2 = (-1, 5, 2)$, $\vec{u}_1 = (4, -2, 1)$ and $\vec{u}_2 = (2, -3, 1)$ we find the distance:

$$d = \frac{|\,(\vec{r}_2 - \vec{r}_1) \cdot (\vec{u}_1 \times \vec{u}_2)\,|}{|\,\vec{u}_1 \times \vec{u}_2\,|} = \frac{|\,(2, -4, -1) \cdot (1, -2, -8)\,|}{|\,(1, -2, -8)\,|} = \frac{18}{\sqrt{69}}.$$

M E T U
Department of Metaphysics

Group	Calculus II MidTerm 2	List No.

Code : *Math 120* Acad. Year : *2004-2005* Semester : *Spring* Instructor : Date : *May 7th,2005* Time : *09:30* Duration : *120 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 6 QUESTIONS ON 5 PAGES TOTAL 60 POINTS

1	2	3	4	5	6	SOLUTION KEY	

Question 1 (10 points) Show that for any tangent plane to the surface $\sqrt{x}+\sqrt{y}+\sqrt{z}=\sqrt{a}$, $(a>0)$ the sum of $x, y,$ and z axis intercepts is constant.
(x_0 is called x intercept if the graph intersects x axis at the point $(x_0, 0, 0)$.)

Solution: Let $f(x,y,z)=\sqrt{x}+\sqrt{y}+\sqrt{z}-\sqrt{a}$ then the surface , S, is given by $f(x,y,z)=0$. Let the point $P_0(x_0,y_0,z_0)\in S$ be the point of tangency. Then

$$\vec{\nabla} f(x,y,z)=\frac{1}{2\sqrt{x}}\vec{i}+\frac{1}{2\sqrt{y}}\vec{j}+\frac{1}{2\sqrt{z}}\vec{k}$$

and hence

$$\vec{n}=\vec{\nabla} f(x_0,y_0,z_0)=\frac{1}{2\sqrt{x_0}}\vec{i}+\frac{1}{2\sqrt{y_0}}\vec{j}+\frac{1}{2\sqrt{z_0}}\vec{k}$$

is a normal vector to the tangent plane. Therefore, the equation of the tangent plane is obtained as

$$\frac{1}{2\sqrt{x_0}}(x-x_0)+\frac{1}{2\sqrt{y_0}}(y-y_0)+\frac{1}{2\sqrt{z_0}}(z-z_0)=0.$$

Now,
$x-$intercept is obtained, for $y=z=0$, as: $(\sqrt{x_0}+\sqrt{y_0}+\sqrt{z_0})\sqrt{x_0}$
$y-$intercept is obtained, for $x=z=0$, as: $(\sqrt{x_0}+\sqrt{y_0}+\sqrt{z_0})\sqrt{y_0}$
$z-$intercept is obtained, for $x=y=0$, as: $(\sqrt{x_0}+\sqrt{y_0}+\sqrt{z_0})\sqrt{z_0}$. Therefore the sum of intercepts is:

$$(\sqrt{x_0}+\sqrt{y_0}+\sqrt{z_0})^2=(\sqrt{a})^2=a$$

which is a constant.

Question 2 (12 points)

Consider the function $f(x,y)=\dfrac{x^3}{x^2+y^2}$ if $(x,y)\neq(0,0)$, $\quad f(0,0)=0$. Show that;

a) f is continuous everywhere

Solution: We need to check only continuity at the point (0, 0).

$$0\leq\left|\frac{x^3}{x^2+y^2}\right|=\left|\frac{x^2.x}{x^2+y^2}\right|\leq|x|$$

so by the squeeze theorem, we have

$$\lim_{(x,y)\to(0,0)}f(x,y)=\lim_{(x,y)\to(0,0)}\frac{x^3}{x^2+y^2}=0=f(0,0)$$

meaning that f is continuous at (0, 0).

b) all directional derivatives of f **exist** at $(0,0)$, using limit definition of directional derivative

Solution: Let $\vec{u}=a\vec{i}+b\vec{j}$ with $a^2+b^2=1$. Then

$$D_{\vec{u}}f(0,0)=\lim_{h\to0}\frac{f(ah,bh)-f(0,0)}{h}=\lim_{h\to0}\frac{a^3h^3/[h^2(a^2+b^2)]-0}{h}=a^3$$

which completes the proof.

c) f is **not** differentiable $(0,0)$

$$\begin{aligned}\lim_{(h,k)\to(0,0)}\frac{f(h,k)-f(0,0)-hf_x(0,0)-kf_y(0,0)}{\sqrt{h^2+k^2}} &= \lim_{(h,k)\to(0,0)}\frac{f(h,k)-h}{\sqrt{h^2+k^2}}\\ &= \lim_{(h,k)\to(0,0)}\frac{\frac{h^3}{h^2+k^2}-h}{\sqrt{h^2+k^2}}\end{aligned}$$

along the line $k=h$ we have

$$\lim_{h\to0}\frac{-h/2}{\sqrt{2}|h|}$$

which does not exist. Hence, f is not differentiable at $(0,0)$.

d) find the set of all vectors $\vec{v}$ for which the equation $D_{\vec{v}}f(0,0)=\nabla f(0,0)\cdot\vec{v}$ is satisfied.

Solution: Let $\vec{v}=v_1\vec{i}+v_2\vec{j}$. Then, by (b) we have $D_{\vec{v}}f(0,0)=v_1^3$ and $\vec{\bigtriangledown}f(0,0)=\vec{i}$. Therefore, $\vec{\bigtriangledown}f(0,0)\cdot\vec{v}=v_1$. Now, the required condition is $v_1=v_1^3$ which gives us $v_1=0, v_1=\pm1$. For, $v_1=0$ we have $v_2=\pm1$ and for $v_1=\pm1$ we have $v_2=0$. Thus, the directions are $\pm\vec{i}$ and $\pm\vec{j}$.

Question 3 (8 points) Let $z = f(x, y) = \sqrt{20 - x^2 - 7y^2}$. Use Tangent Plane Approximation to approximate $f(1.95, 1.08)$.

Solution: We have

$$f_x(x, y) = \frac{-x}{\sqrt{20 - x^2 - 7y^2}}, \quad f_y(x, y) = \frac{-7y}{\sqrt{20 - x^2 - 7y^2}}.$$

So $f_x(2, 1) = -2/3$ and $f_y(2, 1) = -7/3$. Therefore

$$\begin{aligned} f(1.95, 1.08) &\approx f(2, 1) + f_x(2, 1)(1.95 - 2) + f_y(2, 1)(1.08 - 1) \\ &= 3 + (-2/3)(-0.05) + (-7/3)(0.08) \\ &= 3 + \frac{0.1 - 0.56}{3}. \end{aligned}$$

Question 4 (10 points) Given that the function $z = f(\frac{x}{y}, \frac{y}{x})$ has continuous partial derivatives in the neighborhood of the point $(x, y) = (1, -1)$ with
$f_1(-1, -1) = 2 = f_2(1, -1)$, $\quad f_1(1, -1) = -2 = f_2(-1, -1)$,
$f_{11}(-1, -1) = 6 = f_{12}(1, -1)$, $\quad f_{11}(1, -1) = 8 = f_{21}(1, -1)$,
$f_{12}(-1, -1) = -5 = f_{21}(-1, -1)$, $\quad f_{22}(1, -1) = 1 = f_{22}(-1, -1)$.
Find $z_{12} = \frac{\partial^2 z}{\partial y \partial x}$ at the point $(x, y) = (1, -1)$.

Solution: Let $s = x/y, t = y/x$ then $(x, y) = (1, -1)$ gives $(s, t) = (-1, -1)$ and $z = f(x/y, y/x) = f(s, t)$. Now,

$$z_1 = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial z}{\partial t}\frac{\partial t}{\partial x} = \frac{1}{y} f_1(s, t) - \frac{y}{x^2} f_2(s, t).$$

Similarly,

$$\begin{aligned} z_{12} &= \frac{\partial}{\partial y}\left[\frac{1}{y} f_1(s, t) - \frac{y}{x^2} f_2(s, t)\right] \\ &= \frac{-1}{y^2} f_1(s, t) + \frac{1}{y}\left[f_{11}(s, t)\frac{\partial s}{\partial y} + f_{12}(s, t)\frac{\partial t}{\partial y}\right] \\ &\quad -\frac{1}{x^2} f_2(s, t) - \frac{y}{x^2}\left[f_{21}(s, t)\frac{\partial s}{\partial y} + f_{22}(s, t)\frac{\partial t}{\partial y}\right] \\ &= \frac{-1}{y^2} f_1(s, t) + \frac{1}{y}\left[f_{11}(s, t)\frac{-x}{y^2} + f_{12}(s, t)\frac{1}{x}\right] \\ &\quad -\frac{1}{x^2} f_2(s, t) - \frac{y}{x^2}\left[f_{21}(s, t)\frac{-x}{y^2} + f_{22}(s, t)\frac{1}{x}\right] \end{aligned}$$

Put $x = 1, y = -1$ to get

$$\begin{aligned} z_{12}(1, -1) &= -f_1(-1, -1) - [-f_{11}(-1, -1) + f_{12}(-1, -1)] \\ &\quad -f_2(-1, -1) + [-f_{21}(-1, -1) + f_{22}(-1, -1)] \\ &= 17. \end{aligned}$$

Question 5 (10 points) Use the Method of Lagrange Multipliers to find the least and greatest distance between the origin and the ellipse $17x^2 + 12xy + 8y^2 - 100 = 0$.

Solution: Let the point be (x, y) then the distance is $d = \sqrt{x^2 + y^2}$. Now, we define

$$L(x, y, \lambda) = x^2 + y^2 + \lambda(17x^2 + 12xy + 8y^2 - 100)$$

$$\frac{\partial L}{\partial x} = 2x + \lambda(34x + 12y) = 0 \quad (1)$$

$$\frac{\partial L}{\partial y} = 2y + \lambda(12x + 16y) = 0 \quad (2)$$

$$\frac{\partial L}{\partial \lambda} = 17x^2 + 12xy + 8y^2 - 100 = 0 \quad (3)$$

From the equations (1) and (2) we write $\lambda = \frac{-x}{17x + 6y}$ and $\lambda = \frac{-y}{6x + 8y}$, respectively. Equating them and performing cross multiplication we get

$$2x^2 - 3xy - 2y^2 = 0. \quad (4)$$

Multipying, (4) by 4, and adding with (3) we obtain, $25x^2 - 100 = 0$ or $x = \pm 2$.
For $x = 2$, (4) becomes $8 - 6y - 2y^2 = 0$ which has the roots $y = 1$ and $y = -4$.
For $x = -2$, (4) becomes $8 + 6y - 2y^2 = 0$ which has the roots $y = -1$ and $y = 4$. Therefore we have $P_1(2, 1), P_2(2, -4), P_3(-2, -1), P_4(-2, 4)$.

Therefore, the least distance is $\sqrt{5}$ and the greatest distance is $2\sqrt{5}$.

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Question 6 (10 points) Find and classify all the critical points of the function $f(x,y) = \frac{1}{3}x^3 + xy^2 - x$.

Solution: We first find the critical points:

$$f_1(x,y) = x^2 + y^2 - 1 = 0 \tag{5}$$

$$f_2(x,y) = 2xy = 0 \tag{6}$$

The equation in (6) gives us $x = 0$ or $y = 0$. If $x = 0$, (5) implies that $y = \pm 1$ and if $y = 0$, (5) implies that $x = \pm 1$. Therefore, we have $P_1(0,1), P_2(0,-1), P_3(1,0), P_4(-1,0)$ as critical points.

$$f_{11}(x,y) = 2x, \quad f_{12}(x,y) = f_{21}(x,y) = 2y, \quad f_{22}(x,y) = 2x$$

$$\Delta = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} = \begin{vmatrix} 2x & 2y \\ 2y & 2x \end{vmatrix} = 4(x^2 - y^2)$$

and

$\Delta(P_1) = -4 < 0$ so P_1 is a SADDLE point,

$\Delta(P_2) = -4 < 0$ so P_2 is a SADDLE point,

$\Delta(P_3) = 4 > 0$ and $f_{11}(P_3) = 2 > 0$ so P_3 is a LOCAL MIN point,

$\Delta(P_4) = 4 > 0$ and $f_{11}(P_4) = -2 < 0$ so P_4 is a LOCAL MAX point.

M E T U
Department of Mathematics

Group	Calculus II Final Exam	List No.

Code : *Math 120* Acad. Year : *2004-2005* Semester : *Spring* Instructor : Date : *June 1st,2005* Time : *09:30* Duration : *150 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 6 QUESTIONS ON 6 PAGES TOTAL 90 POINTS

1	2	3	4	5	6	7	8		

Question 1 (15 points) Let $\Re$ be the region in the first quadrant of the $xy-$plane bounded by the curves $xy = 4,\ xy = 8,\ xy^3 = 5$ and $xy^3 = 15$. Evaluate the double integral $\displaystyle\int_{\Re}\int xy^3\cos(xy)dxdy$

Question 2 (15 points) Consider a sequence $\{a_n\}$ such that $a_n > 0$ for all $n \geq 1$, $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 2$ and the series $\sum \frac{a_n}{2^n}$ diverges

a)Determine the interval of convergence (with end points) of the series $f(t) = \sum_{n=1}^{\infty} a_n t^{2n}$.

b)Determine the largest open domain in R^2 in which the function $g(x, y) = f(x) + yf(y)$ has a differential.

c)Show that the only critical point of $g(x, y)$ is the origin.

d)Determine the nature of the critical point $(0, 0)$.

Question 3 (15 points) If C is the intersection of $z = \ln(1+x)$ and $y = x$ from $(0,0,0)$ to $(1,1,\ln 2)$, evaluate $\int_C [(2x\sin(\pi y) - e^z)dx \; + \; (\pi x^2\cos(\pi y) - 3e^z)dy \; + \; xe^z dz]$

Question 4 (15 points) Find the absolute extrema of the function $f(x, y) = xye^{-x-y}$ on the triangular region with verticies $(0, 0)$, $(0, 4)$ and $(4, 0)$.

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Question 5 (15 points) Using Implicit Function Theorem show that the equations $u = x^2 - y^2, \quad v = 2xy$ define x and y implicitly as functions of u and v for values of (x, y) near $(1, 1)$. Find $\dfrac{\partial x}{\partial u}$ at $(u, v) = (0, 2)$.

Question 6 (15 points) Find the volume lying between the paraboloids $z = x^2 + y^2$ and $3z = 4 - x^2 - y^2$.